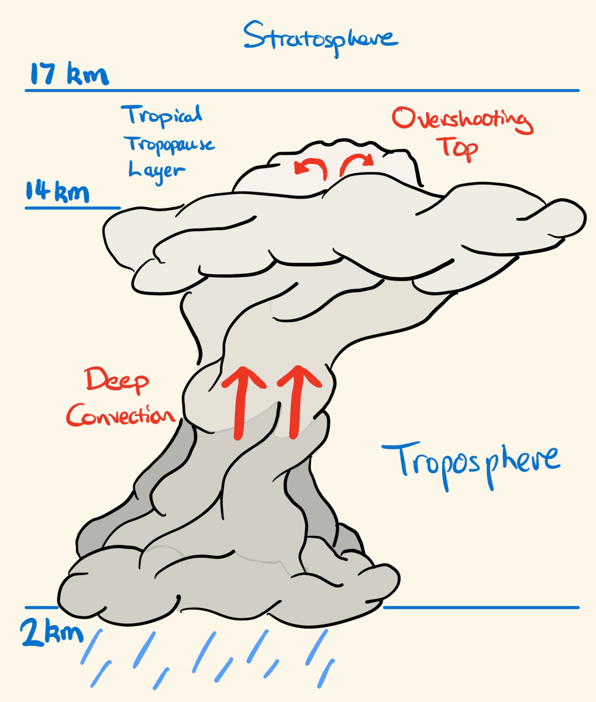
Diagnosing tracer transport in convective penetration of a stably stratified layer

Charles Powell, Peter Haynes, John Taylor

DAMTP, University of Cambridge

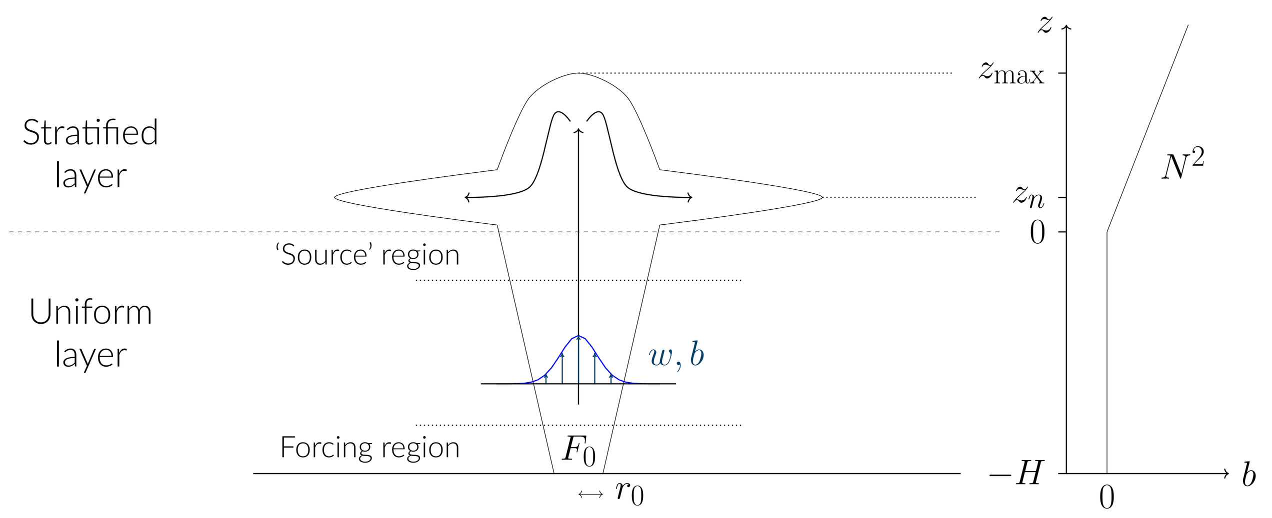
Background

- We consider a buoyant plume, carrying a passive tracer, penetrating from an unstratified region into a strongly stably stratified layer.
- This is an idealised representation of the tropical tropopause layer, where convective overshoots generated by strong thunderstorm complexes can penetrate into the lower stratosphere.
- The net transport of water vapour by these overshoots is potentially important (Fueglistaler et al., 2009; Jensen et al., 2007), but detailed numerical simulation of entire thunderstorm complexes remains computationally expensive (Dauhut et al., 2015, 2018).



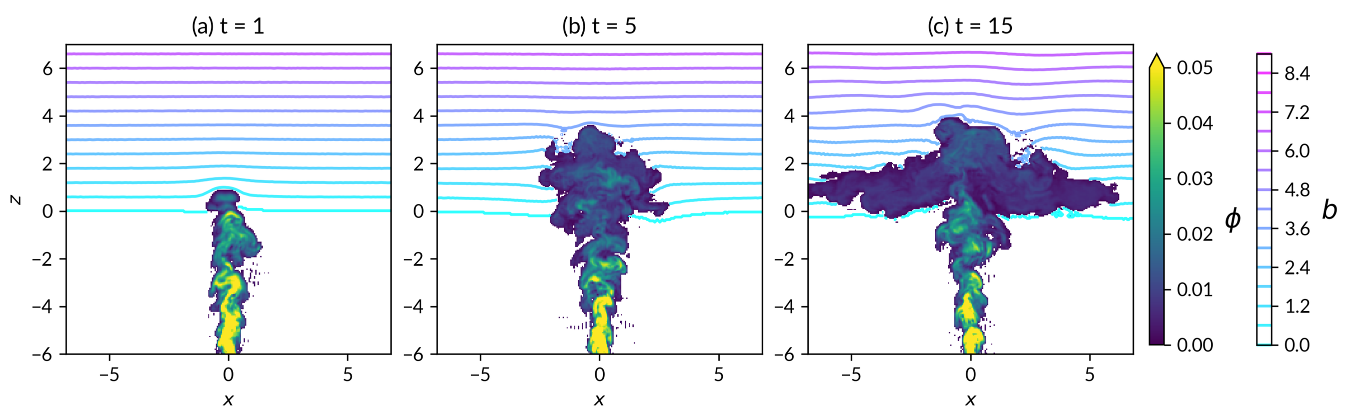
Idealised numerical simulations

- We perform large eddy simulations of the Boussinesq Navier-Stokes equations in a doubly periodic domain.
- A plume with a fixed integral source buoyancy flux F_0 and source radius r_0 is generated by relaxing the buoyancy and vertical velocity towards the analytic profiles of an axisymmetric forced plume, in a thin forcing region at the bottom of a uniform layer of depth H. The plume carries a passive tracer with concentration $\phi(\boldsymbol{x}, t)$ and the buoyancy field is $b(\boldsymbol{x}, t)$.
- A linear stably stratified layer with constant buoyancy frequency N lies above the uniform layer. All quantities shown are non-dimensionalised using F_0 and N. The characteristic lengthscale is $F_0^{1/4}N^{-3/4}$ and the characteristic timescale is N^{-1}.



Plume dynamics

- As the plume penetrates the stably stratified layer and encounters more buoyant fluid, it decelerates and overturns at $z_{\max}$. Plume fluid then subsides to the equilibrium height z_n and forms a radially-spreading intrusion.
- Irreversible transport of tracer to a given buoyancy surface depends on the environmental buoyancy accessed via mixing during the transient rise of plume fluid deep into the stratified layer. All plots and data shown are from a simulation with $N = 1\,\mathrm{s}^{-1}$ and $F_0 = 2.3 \times 10^{-7}\,\mathrm{m}^4\mathrm{s}^{-3}$.



Buoyancy-tracer volume distribution

- The joint distribution $W(b, \phi; t)$ of volume in (b, ϕ)-space applied to a fixed volume V is defined as

$$W(b', \phi'; t) = \int_V \delta(b(\boldsymbol{x}, t) - b')\delta(\phi(\boldsymbol{x}, t) - \phi')\,\mathrm{d}V, \tag{1}$$

such that $W(b', \phi'; t)\,\mathrm{d}b\,\mathrm{d}\phi$ is the volume of fluid within V with buoyancy between b' and $b' + \mathrm{d}b$ and tracer concentration between ϕ' and $\phi' + \mathrm{d}\phi$. Here, V is the stratified layer $z \geq 0$. The volume distribution W satisfies the evolution equation:

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\nabla_{(b', \phi')} \cdot \boldsymbol{F} - S \tag{2}$$

where the flux $\boldsymbol{F}$ is

$$\boldsymbol{F}(b', \phi'; t) = \int_V \kappa(\nabla^2 b, \nabla^2 \phi)\,\delta(b - b')\delta(\phi - \phi')\,\mathrm{d}V, \tag{3}$$

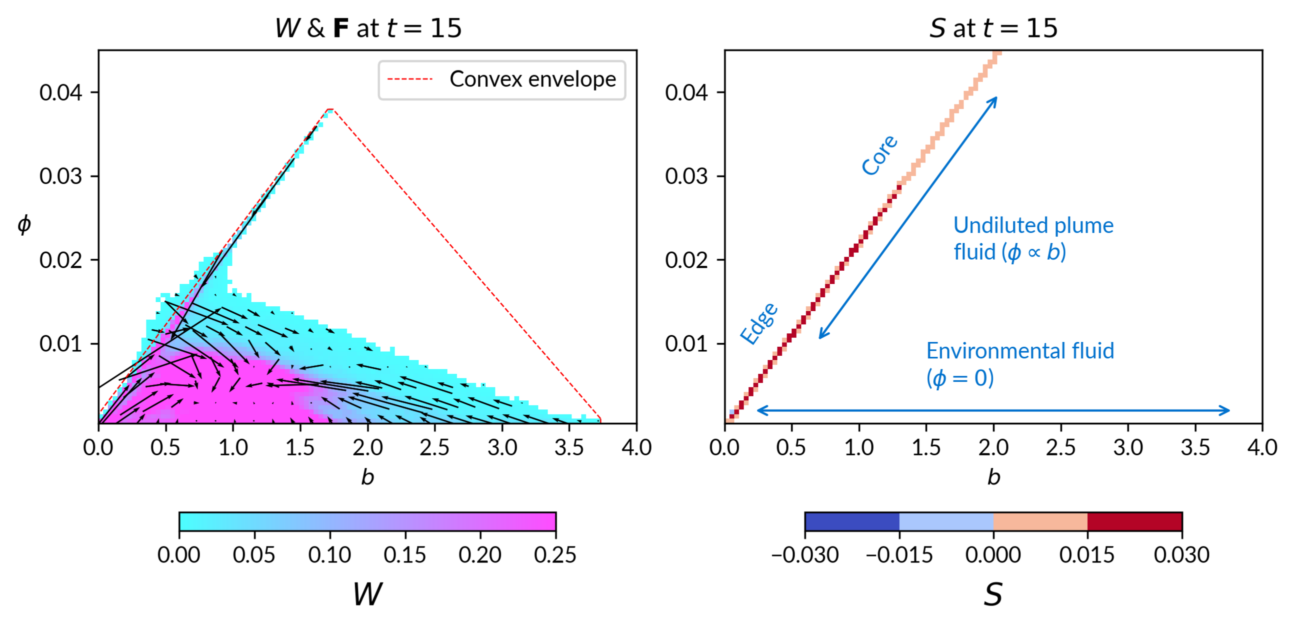
and the source/sink S is

$$S(b', \phi'; t) = \int_{\partial V} \boldsymbol{u} \cdot \boldsymbol{n}\,\delta(b - b')\delta(\phi - \phi')\,\mathrm{d}A, \tag{4}$$

where $\boldsymbol{u}$ is the fluid velocity in physical space and $\boldsymbol{n}$ is the outward normal.
- The volume distribution W within V is independent of advection; only fluid entering V and diffusive mixing result in changes to W.
- A mixture of fluid parcels lies in the (b, ϕ)-space convex envelope of those parcels.
- The cumulative mixed volume distribution $M(b, \phi; t)$ is the change in the volume distribution W compared with the cumulative source distribution up to time t,

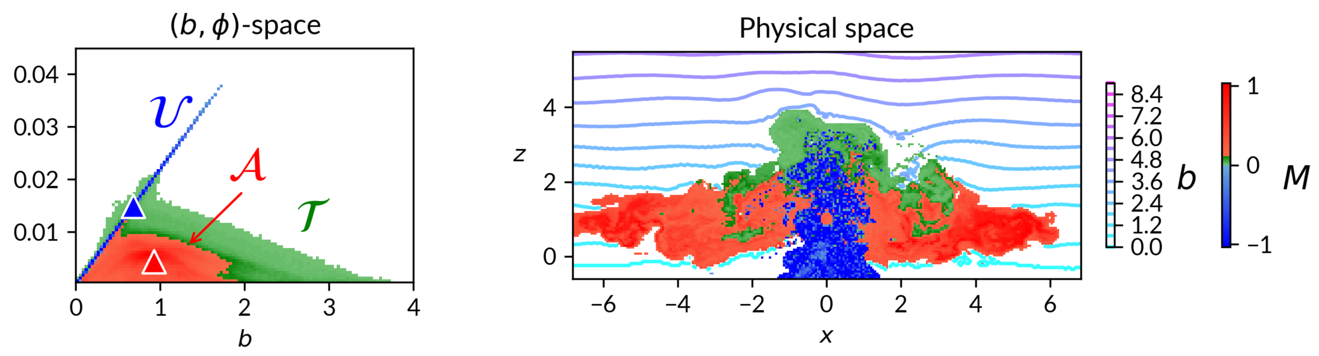
$$M(b', \phi'; t) = W(b', \phi'; t) - \int_0^t -S(b', \phi'; t')\,\mathrm{d}t' = -\int_0^t \nabla_{(b', \phi')} \cdot \boldsymbol{F}(b', \phi'; t')\,\mathrm{d}t'. \tag{5}$$

Volume distribution in convective penetration



- $S(b, \phi; t)$ shows that undiluted plume fluid entering the stratified layer lies along a line in (b, ϕ)-space. Turbulent mixing during the rise through the uniform layer gives b and ϕ Gaussian radial profiles with the same characteristic width, so $\phi \propto b$. At the edges of the plume, b and ϕ are small, whilst b and ϕ are maximised near the plume centreline.
- The environmental fluid surrounding the plume enters the volume distribution from the $\phi = 0$ axis in (b, ϕ)-space. Mixtures of plume and environmental fluid lie within the convex envelope of the source line and the $\phi = 0$ axis up to the environmental buoyancy corresponding to $z_{\max}$.
- The convex envelope is not filled: extreme values of b and ϕ in the plume core do not mix with the environment due to shielding by the plume edges and intrusion, until overturning near $z_{\max}$ when b and ϕ have been reduced by mixing.

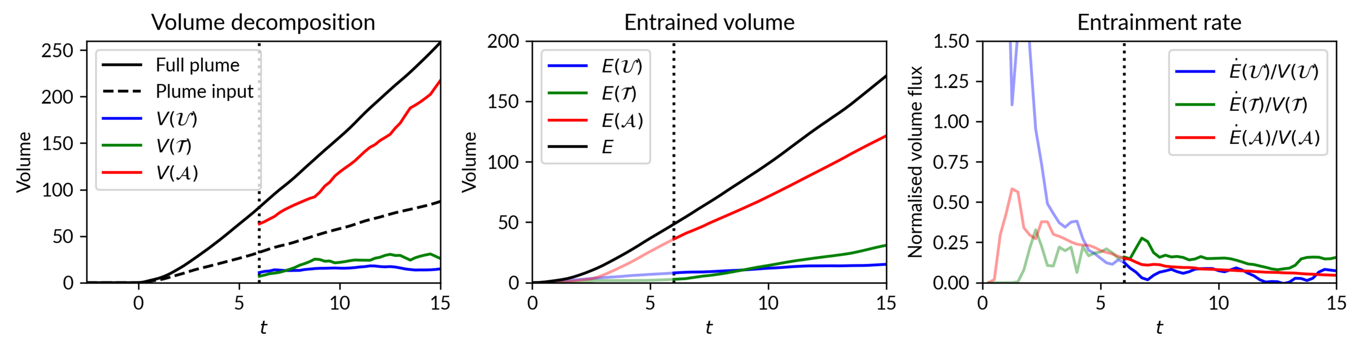
Input, transport and accumulation regions



- The distribution $M(b, \phi; t)$ represents the integrated effect of diffusive mixing up to time t; where $M < 0$ there is a net loss of volume locally in (b, ϕ)-space and where $M > 0$ there is a net gain of volume due to mixing.
- In quasi-steady state, we use M to partition (b, ϕ)-space into three regions, each corresponding with coherent regions of physical space.
- Region $\mathcal{U}$ where $M < 0$ corresponds to undiluted plume fluid in the plume core, which continuously enters the stratified layer and moves away from the source line in (b, ϕ)-space due to mixing with the environment.
- Region $\mathcal{T}$ where $M > 0$ is small and non-increasing corresponds with the plume cap, where much of the undiluted plume fluid is first exposed to, and mixes with, the surrounding environment with significantly larger buoyancy.
- Region $\mathcal{A}$ where $M > 0$ is large and increasing corresponds with the intrusion, where significant volume accumulates as mixtures of environmental and plume fluid become well-mixed and entrain further environmental fluid surrounding the intrusion.
- Formally, regions $\mathcal{A}$ and $\mathcal{T}$ are separated by a threshold $M = m(t)$ which minimises the total volume change in region $\mathcal{T}$:

$$\int_{\{0<M\leq m(t)\}} \nabla_{(b,\phi)} \cdot \boldsymbol{F}\,\mathrm{d}b\mathrm{d}\phi \tag{6}$$

Entrainment



- The volume of the plume cap and undiluted plume fluid is approximately constant, whilst the volume of the intrusion grows proportional to the volume of the full plume.
- The volume of environmental fluid entrained by each region is calculated by integrating the boundary flux F_ϕ across the intersection of each region with the $\phi = 0$ axis.
- The intrusion entrains the largest volume of environmental fluid as it spreads radially.
- However, the entrainment rate, defined as the rate of change of the entrained volume as a fraction of the region volume, is largest in the plume cap.

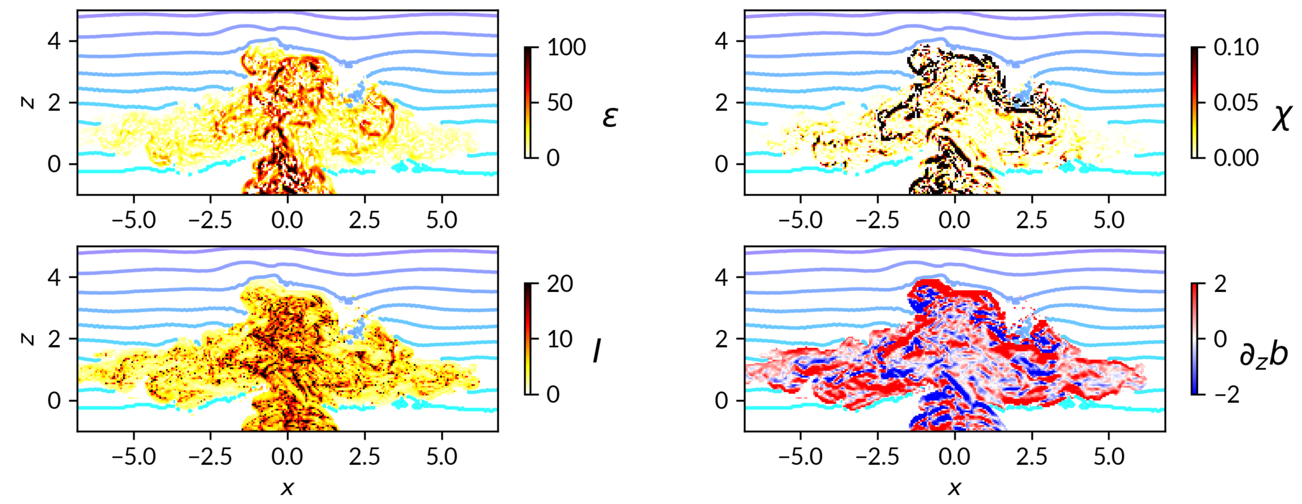
Mixing diagnostics

- To examine the mixing in each region identified by M, we consider the spatial distribution of four mixing diagnostics, shown here as cross-sections in the $x - z$ plane and as volume averages in each region $\mathcal{U}, \mathcal{T}$ and $\mathcal{A}$:
  - turbulent kinetic energy dissipation rate ε
  - potential energy dissipation rate χ
  - local vertical buoyancy gradient $\partial_z b$
  - activity parameter $I = (\varepsilon/\nu)/\partial_z b$ which quantifies the balance between the timescale for turbulence to develop and the local buoyancy timescale

	Full plume	$\mathcal{U}$	$\mathcal{T}$	$\mathcal{A}$
Volume %	100	4.27	10.3	85.5
$\overline{I}$	31.2	242	37.9	19.6
$\langle\lvert\partial_z b\rvert\rangle$	1.23	0.99	2.6	1.08
$\overline{\varepsilon}$	$4.14 \cdot 10^{-2}$	0.29	0.12	$1.95 \cdot 10^{-2}$
$\overline{\chi}$	$2.6 \cdot 10^{-2}$	$3.43 \cdot 10^{-2}$	0.14	$1.2 \cdot 10^{-2}$
η	0.39	0.11	0.54	0.38

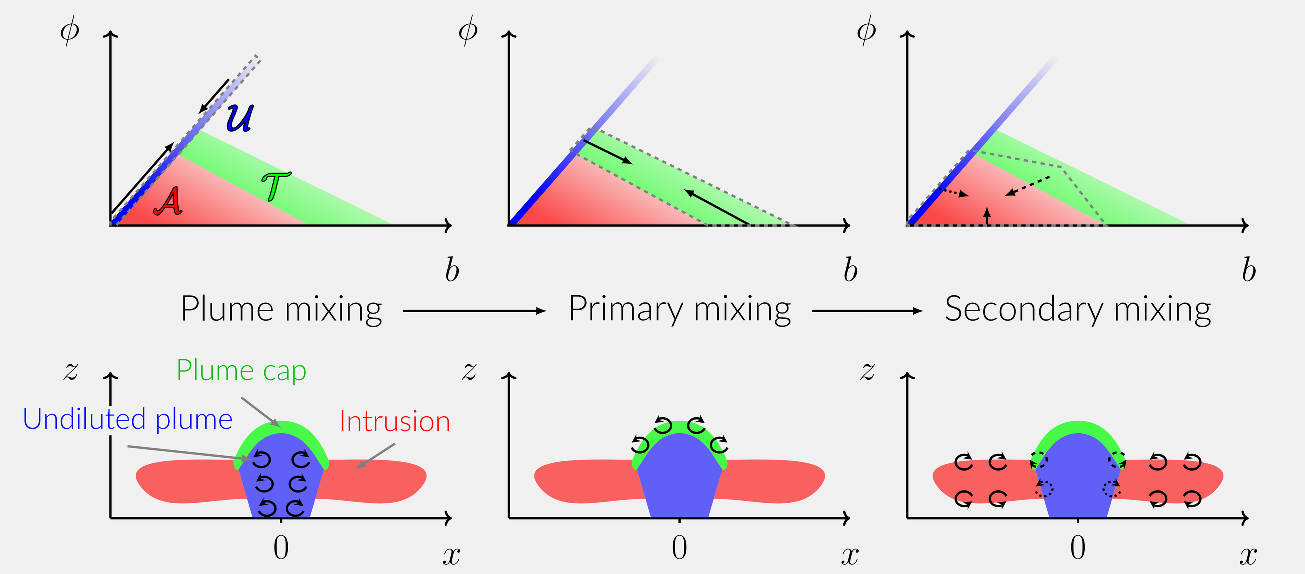
Table 1. Mixing diagnostics at $t = 15$.

- The instantaneous mixing efficiency is calculated as $\eta = \overline{\chi}/\left(\overline{\chi} + \overline{\varepsilon}\right)$, which quantifies the fraction of energy dissipated via turbulence that leads to diffusive mixing.



Stages of mixing

- Undiluted plume fluid rises upwards, with large TKE dissipation ε and relatively weak buoyancy gradients. Mixing in this region exhibits low efficiency, around 10%.
- As fluid overturns and impinges on the surrounding environmental fluid, intense buoyancy gradients are established as indicated by a thin layer of very large χ in the plume cap. Here, mixing is the most efficient, around 50%.
- In the spreading intrusion, there is continued efficient mixing as fluid homogenises and entrains environmental fluid at the bottom of the stratified layer, but buoyancy gradients and turbulent dissipation are both weak.



Summary

- We provide a framework for quantifying the mixing effect of a buoyant plume with a passive tracer as it penetrates into a stably stratified environment. In particular, we identify a transport region in which much of the transition from the undiluted plume to mixed intrusion takes place.
- Future work is focused on establishing the dependence of these results on F_0, N, and the configuration of the plume at penetration – offering a possible approach to parameterisations of tracer transport and mixing in convective penetration.
- By introducing large-scale vertical shear and buoyancy-dependent tracer concentration, which captures the effect of water vapour saturation, we can investigate setups closer to the atmospheric problem.

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Acknowledgements

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This work has been submitted to J. Fluid Mech. View the pre-print by scanning the QR code or going to:
`https://arxiv.org/abs/2310.06096`